



UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

SCHOOL OF ENGINEERING

MAIN EXAMINATIONS: NOVEMBER 2025

COURSE AND CODE: PROCESS MODELLING AND OPTIMIZATION ENCH3PO

DURATION: 3 HOURS

TOTAL MARKS: 100

INTERNAL EXAMINER/S:

DR S MOODLEY
DR S FASHU

INTERNAL MODERATOR:

PROF K MOODLEY

EXTERNAL MODERATOR:

MR P BIYELA

INSTRUCTIONS:

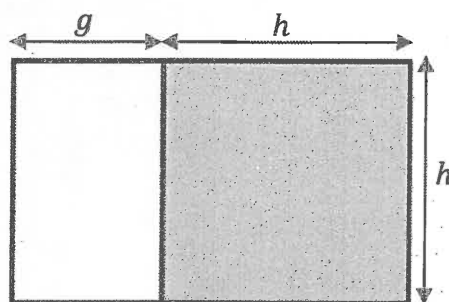
1. ALL questions should be attempted. Answer in INK.
2. Any programmable or non-programmable calculator may be used provided it has been cleared of any information that would subvert the purpose of the examination.
3. Calculations must be shown in sufficient detail to illustrate your understanding of the procedure.
4. Two answer books are provided. Label them Book A and Book B. Answer Section A in Book A and Section B in Book B.
5. This examination question paper, together with all associated diagrams, **MUST BE HANDED IN TOGETHER WITH YOUR SCRIPT.**

Student Number:

220039712

SECTION A**QUESTION 1**

- (a) Explain what is meant by a real-valued function $f(x): \mathbb{R} \rightarrow \mathbb{R}$ being 'unimodal' in an interval $a \leq x \leq b$. (1)
- (b) Use the areas of the rectangles and square shown below to calculate the value of the so-called 'golden ratio' (denoted by ϕ).



- (c) Explain the computational significance (specifically, the advantage) of using the golden ratio instead of, say, a numerical value of 1.5 when calculating the interior points (c and d) in subsequent intervals when using the golden section search (GSS) method for optimization. (3)
- (d) The total entropy of mixing of a non-ideal binary mixture composed of chemical 1 and chemical 2 using the 3-suffix Margules activity coefficient model is

$$\Delta S_{mix} = -n_{tot}R[x_1 \ln(x_1) + x_2 \ln(x_2) - x_1 x_2 (A_{21}x_1 + A_{12}x_2)]$$

where n_{tot} is the total number of moles, R is the universal (ideal) gas constant (in SI units), x_1 is the mole fraction of chemical 1, x_2 is the mole fraction of chemical 2 while A_{12} and A_{21} are the activity coefficient model constants.

If $n_{tot} = 10$ moles, $A_{21} = -3$ and $A_{12} = 1$, determine the mole fraction of chemical 1 that will maximize the entropy of mixing of the solution. Use the GSS method with an initial interval $x_1 = [0.05, 0.95]$. Perform one iteration only and determine the four points in the new interval that would have been used in the second iteration.

Hint: Maximizing a single-variable function is equivalent to minimizing the same function multiplied by minus (negative) one.

(8)

TOTAL /15/

QUESTION 2

Consider a solid (i.e., not hollow) sphere of radius R , emissivity ε and constant thermal conductivity k . The sphere is giving off heat to its surroundings (which is at a constant temperature T_∞) via radiation only, at a known radiative flux q''_{rad} . Within the sphere, heat is generated at a uniform volumetric rate $\dot{q}_V$ (in W/m^3).

- (a) Derive the ordinary differential equation (ODE) that describes the steady-state radial temperature distribution within the sphere, by using a differential (shell) energy balance. The coefficient of the highest order derivative in the ODE **must** be equal to unity (i.e., one). Also, write down the boundary conditions.

Hints: For the $r = 0$ boundary condition, use symmetry. For the $r = R$ boundary condition, the surface temperature can be solved for by using the radiative flux.

N.B.: **NO marks** will be awarded if you use the heat equation PDE i.e.,

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{q}_V, \text{ as that would NOT constitute a differential energy balance.}$$

(13)

- (b) Hence, calculate the temperature distribution within the sphere, given the following data:

Property or variable	Value	Unit(s)
Δr	0.1	m
R	0.3	m
ε	0.95	dimensionless
k	0.2	$\text{W.m}^{-1}.\text{K}^{-1}$
T_∞	300	K
q''_{rad}	100	W.m^{-2}
$\dot{q}_V$	1000	W.m^{-3}
σ	5.67×10^{-8}	$\text{W.m}^{-2}.\text{K}^{-4}$

(14)

Supporting Information:

$$\frac{d^2 y}{dx^2} \approx \frac{(y_{i-1} - 2y_i + y_{i+1}))}{(\Delta x)^2}$$

$$\frac{dy}{dx} \approx \frac{(y_{i+1} - y_{i-1}))}{2\Delta x}$$

TOTAL /27/

QUESTION 3

Use Newton's method for multivariable optimization to minimize the function

$$f(x, y) = 2x^2 + 3y^2 - 3xy - 10$$

Use $P_0 = (x_0, y_0) = (-2, 2)$, and perform one iteration only (i.e., calculate P_1).

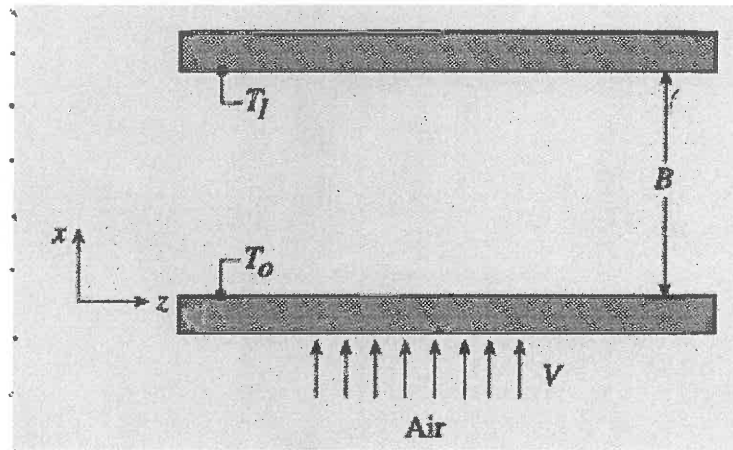
Supporting Information:

$$P_{i+1} = P_i - H_i^{-1} \nabla f_i$$

TOTAL /8/

SECTION B**QUESTION 4**

Two large porous plates are separated by a distance B as shown in the figure. The temperatures of the lower and the upper plates are T_0 and T_1 , respectively, with $T_1 > T_0$. Air at a temperature of T_0 is blown in the x -direction with a velocity of v .



(a) Derive the energy equation.

(6)

(b) Introduce the following dimensionless variables to nondimensionalize the energy equation and the boundary conditions.

$$\theta = \frac{T - T_0}{T_1 - T_0}$$

$$\xi = \frac{x}{B}$$

$$\lambda = \frac{\rho C_P v B}{k}$$

(6)

(c) Solve the non-dimensional energy equation for θ .

(8)

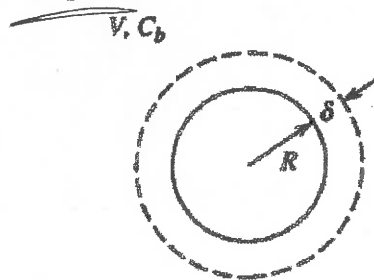
(d) Find the heat flux at the lower plate.

(5)

TOTAL /25/

QUESTION 5

Dissolution of a solid particle in a finite volume (e.g., dissolution of a medicine tablet in the stomach) may be considered as a kinetic process with the existence of a layer adjacent to the solid surface and the diffusion through this layer being the controlling step to the dissolution process as shown in the figure. The dissolved material is then consumed by the stomach according to a first order chemical reaction. Assume the diffusion coefficient D is constant, at any given time the concentration on the solid surface is C_0 and the concentration at the end of this layer i.e. of the stomach is C_b .



- (a) The thickness of the surface layer around the solid particle is taken as δ . Assuming the particle is spherical in shape, derive the mass balance equation for the dissolved species in this layer (see Fig above). Then show that under quasi-steady-state conditions the mass balance equation is

$$\frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) = 0 \quad (4)$$

- (b) What are the boundary conditions?

(2)

- (c) If the density of the particle is ρ and the molecular weight is M , perform the mass balance and show that the particle radius is governed by the following equation

$$\frac{\rho}{M} \frac{dR}{dt} = D \left(\frac{\partial C}{\partial r} \right)_{r=R} \quad (5)$$

- (d) If the consumption of the dissolved species in the finite volume is very fast, that is, $C_b \approx 0$.

- (i) Solve the mass balance equation for the dissolved species in the surface layer in part (a) and the particle radius equation in part (c) to derive a solution for the particle radius as a function of time and

(12)

- (ii) Determine the time it takes to dissolve completely the solid particle.

(2)

Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$	$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s} \quad (s > 0)$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2} \quad (s > 0)$
t^n	$\frac{n!}{s^{n+1}} \quad (s > 0)$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2} \quad (s > a)$
e^{at}	$\frac{1}{s-a} \quad (s > a)$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2} \quad (s > a)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad (s > a)$	$y'(t)$	$sY(s) - y(0)$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2} \quad (s > 0)$	$y''(t)$	$s^2 Y(s) - sy(0) - y'(0)$